

Learning from a rival bank and lending boom

Yoshiaki Ogura^{*}

Institute of Economic Research, Kyoto University, Yoshida Honmachi Sakyo-ku, Kyoto 606-8501, Japan

Received 4 February 2005

Available online 24 July 2006

Abstract

When bankers observe a rival winning in the interbank competition for lending to a firm, they infer that the firm may be more promising than they had thought. From this consideration, they loosen their creditworthiness tests and lower the interest rates they offer in the next lending competition for the firm. Increased interbank competition reduces the impact of this observational learning and decreases the credit risk taken by each bank because of a severe winner's curse, while it increases the aggregate risk taken by the entire banking sector.

© 2006 Elsevier Inc. All rights reserved.

JEL classification: G21; G28

Keywords: Interbank competition; Financial liberalization; Learning; Winner's curse

1. Introduction

What information do banks deduce from rival behavior in a credit market and how do they adjust their lending policy according to this information under competitive pressure? An article titled “Resist Market Pressure to Make ‘Ugly Loans’” in *the American Banker* (Williams, 1998) provides an interesting anecdote. The owner of a chain of franchise stores obtained working capital and long-term financing from a bank. One day, “the borrower received an offer that looked almost too good to be true. A competing bank proposed not only to turn a largely secured loan into a largely unsecured loan, but also to do it for less than the original lender was charging in interest and fees. [...] the original lender felt compelled to match them lest it lose the customer.

^{*} Fax: +81 75 753 7178. Present address: Institute of Economic Research, Hitotsubashi University, 2-1 Naka, Kunitachi, Tokyo 186-8603, Japan

E-mail address: ogura@ier.hit-u.ac.jp.

[...] the new deal was consummated with pricing 45 basis points less, and the annual fee reduced one quarter of a percentage point below the terms of the original loan.”

The theory of informational lock-in (Sharpe, 1990; Rajan, 1992) seems to predict the last phase correctly in the anecdote. After all, the original bank captured the customer by exploiting the informational advantage that it had accumulated in the past relationship. However, in the anecdote, the monopolistic power of the incumbent lender does not seem to be very strong because of the aggressive bidding by a rival. Why could the competing bank make such an aggressive offer? One possible answer is that the rival bank was free riding on the information that was implied in the past lending decision by the incumbent lender. This paper illustrates how the observational learning from rival behaviors drives bankers to an aggressive and seemingly imprudent strategy and how the credit market structure affects the impact of learning.

In another strand of the theory of interbank competition, some articles propose models to analyze a Bertrand competition among banks undertaking imperfect and independent creditworthiness tests of potential borrowers. In these models, each bank is concerned that it might be too optimistic about the creditworthiness of a borrower when it wins in a lending competition since winning implies that the rival knows a more pessimistic side of the borrower (*winner's curse*, Broecker, 1990; Riordan, 1993; Shaffer, 1998). The previous interest rate and the fact that the borrower got a loan in the previous period, however, work for rival banks as a positive signal about the creditworthiness of the borrower if the competition is repeated. The rivals compete more aggressively in the second competition by loosening creditworthiness tests and lowering interest rate bids since the winner's curse decreases as a result of the learning from the previous interest rate. This result provides a rational explanation for the empirical finding that bank officers tend to ease credit standards gradually over an economic boom (Berger and Udell, 2004).¹

A comparative static shows that increased interbank competition, which is expressed by the increase in the number of banks, decreases the impact of observational learning and makes each bank more prudent because of the increased winner's curse, while the aggregate risk taken by the entire banking sector increases. This proposition suggests that financial liberalization results in a lending boom, which increases the aggregate risk taken by the entire banking sector but decreases the risk taken by each individual bank. If a government wants to maintain the aggregate risk at the level before liberalization, the bank supervision must be more conservative, especially after liberalization. These results are consistent with the observation that financial liberalization tends to be followed by a lending boom and crash (Kaminsky and Reinhart, 1999) and another observation that the problem of the malfunctioning bank supervision system tends to be at issue after financial liberalization and a crash (Caprio and Klingebiel, 2000).

In this study, it is assumed that each bank can make, at most, one loan to a firm for the sake of simplicity. This assumption can be interpreted as a consequence of the incentive of each bank to diversify credit risks by avoiding the concentration of risk exposure. In this sense, “a firm” in this model can be interpreted as “a sector” or “an industry” in that a common demand or cost factor affects each firm. If we stick to the interpretation that banks learn about the creditworthiness of each individual firm, then this assumption means that the model in this paper precludes the case in which there exists informational asymmetry among competing banks due to relationship banking. In this case, the results in this paper are principally applicable to interbank competitions in which many de novo banks start to operate after financial liberalization without any exclusive

¹ Easing of credit standards during an economic boom can also occur due to the reputational concerns of banks that have extended loan commitments (see Thakor, 2005).

customer relationships. Major commercial banks in the small business credit market in Japan in the late 1980s and foreign banks in Scandinavian countries in the 1980s and in South East Asia in the 1990s are typical examples of lending booms and crashes following deregulations that brought severe competition in the credit markets in these countries.²

Regarding the signaling effect of a lending decision by a bank, the theoretical and empirical study by [Thakor \(1996\)](#) shows that the positive impact of a lending announcement on the stock return of the borrower is greater when the lender is more capital-constrained. The present study extends the notion of this signaling effect to learning among competing banks and analyzes the impact of the credit market structure to the impact of learning among competing banks. As for the effect of “optimism” in credit markets, [Manove and Padilla \(1999\)](#) have shown that the existence of an optimistic potential entrepreneur, who is boundedly rational in the sense that he does not realize that he is an optimist, causes overlending. [Coval and Thakor \(2005\)](#) show that if entrepreneurs are optimistic and potential financiers are pessimistic, a financial intermediary may be needed to bridge this “beliefs gap.” The present study focuses on the side of lenders and shows the mechanism that makes lenders gradually more optimistic about a borrower’s creditworthiness through a rational price-seeking process.

The rest of the paper is organized as follows. We describe the basic structure of the model in Section 2. We derive the equilibrium in the sequential competitive bidding for a loan and compare the bidding behavior of banks in each stage in Section 3. We present a comparative static about the effect of increased interbank competition in Section 4. The examination of the robustness of the propositions in the previous sections against the modification of some key assumptions is contained in Section 5. The empirical predictions from the model are summarized in Section 6. Section 7 is the conclusion.

2. Model

At date 0, a firm that plans to start a project with costs I applies for a loan to n banks. We assume that the only available financial resource for the firm is bank lending. The revenue from this project is realized two periods later, at date 2. Before then, at date 1, the firm needs to apply for another loan to finance the additional cost I to continue its project at date 1. The revenue from this continuation is realized at date 3. The revenue from the project v is equal to $v_h > I > 0$ in both periods if the project is good, and it is equal to $v_l = 0$ in both periods if the project is bad.

The loan contract that we consider is a standard debt contract. The payoff to a bank from each loan is $\min(v, R) - I$, where R is the gross interest rate. The payoff to a firm is the residual after the repayment, $\max(0, v - R)$. The firm always has an incentive to apply for a loan since it can earn a nonnegative return in any state thanks to the limited liability property of the standard debt contract.

Banks compete for lending to the firm twice, at date 0 and at date 1. A Bertrand competition in a credit market with imperfect creditworthiness tests can be interpreted as a common value first-price auction ([Riordan, 1993](#)). At date 0, each bank has a prior belief about the probability that the firm has a good project: $Prob(v = v_h) = \gamma$. After getting a loan application, Bank i gets

² [Hoshi and Kashyap \(1999\)](#) document that the deregulation in corporate financing instruments in Japan in the 1980s made major corporations independent of commercial banks. This shift urged commercial banks to seek small business loans secured on real estate as a profitable frontier, which eventually resulted in a huge amount of non-performing loans. [Ongena et al. \(2003\)](#) report rapid lending growth after the branching deregulation for foreign banks before the Norwegian banking crisis. [Lauridsen \(1998\)](#) also reports a similar phenomenon in Thailand in the 1990s.

a costless private signal $s_i \in [\underline{s}, \bar{s}]$ about the quality of the project as a result of an imperfect and independent creditworthiness test. s_i is a random draw from a cumulative distribution function, $F(s_i|v)$, which is i.i.d. conditional on v . We assume that the corresponding density function $f(s_i|v)$, which is positive for any $s_i \in [\underline{s}, \bar{s}]$, satisfies:

$$\frac{d}{ds_i} \frac{f(s_i|v_h)}{f(s_i|v_l)} > 0, \quad (1)$$

$$\frac{d}{ds_i} \frac{f(s_i|v_j)}{F(s_i|v_j)} < 0, \quad j = h, l. \quad (2)$$

It is easy to derive the next two inequalities from assumption (1).

$$F(s_i|v_h) < F(s_i|v_l), \quad (3)$$

$$\frac{f(s_i|v_h)}{F(s_i|v_h)} > \frac{f(s_i|v_l)}{F(s_i|v_l)}. \quad (4)$$

Inequality (3) means that a bank is more likely to get a higher signal if the firm has a good project. Assumption (1) assures the informativeness of a private signal s_i .

Each bank Bayesian-updates its belief about the success probability of the potential borrower with the private signal s_i before the first competition. The posterior belief is:

$$\mu(s_i) = \frac{\gamma f(s_i|v_h)}{\gamma f(s_i|v_h) + (1 - \gamma) f(s_i|v_l)}.$$

Each bank bids a gross interest rate $R_1(s_i)$ based on this posterior belief. The firm borrows from the bank that offers the lowest interest rate. We focus on a symmetric equilibrium, in which the bid function of each bank is monotone decreasing in s_i .

We assume that each bank can observe whether the firm could get a loan and the equilibrium interest rate in the first competition. In practice, banks usually ask loan applicants about their past financial transactions, including the interest rates and other terms of their past loans.³ This information is verifiable since the third party can examine the written loan contracts if necessary. However, the interest quotes which did not reach actual loan contracts are not verifiable since there are no written contracts for them. From this consideration, we assume that each bank cannot observe losing bids. Each bank updates its belief by using the additional information reflected in the previous winning bid at the beginning of the second competition and competitively bids again. The timing of the game is summarized in Fig. 1.

For the sake of tractability, we assume that each bank can supply, at most, one loan to the firm.⁴ This assumption can be interpreted as the incentive of banks to limit their exposure to a single risk factor. Under this assumption, only the losers in the first competition bid in the second competition. We will examine the plausibility of this assumption and the robustness of the results against some modifications of this assumption.

³ The information required in a loan application can vary among countries. Interest rates in the past transactions may be unobservable from bankers in some countries. The analysis in the present study is not directly applicable to these countries. However, even if the past interest rates are not directly observable, a bank can roughly calculate the past interest rate from interest costs and repaid amounts, which are usually recorded in the financial statement of each firm.

⁴ By this assumption, the model reduces to a single unit demand sequential auction model (Ortega-Reichert, 1968; Milgrom and Weber, 1982).

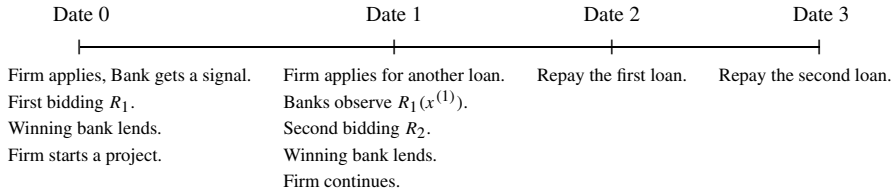


Fig. 1. Flow of the game.

3. Bidding strategy of each bank

3.1. Second competition

We solve the problem backwards. In the second competition, there are two cases with respect to the information environment.

- (1) If one of the competing banks lent in the first competition, each banker's additional information is $x^{(1)}$, which is calculated back from the first-period equilibrium interest rate $R_1(x^{(1)})$, which is strictly decreasing in $x^{(1)}$, as we will show in the next section. $x^{(1)}$ is the highest signal among the private signals of rivals. In addition, the number of rivals decreases by 1.
- (2) If nobody lent in the first competition, the additional information for each bank is the fact that every signal was less than the screening threshold s_{*1} , which will be derived in a later section. The number of bidders does not change in this case.

3.1.1. One of the competing banks lent in the previous period

In this case, the firm applies for loans to $n - 1$ banks who lost in the first competition at date 1. Each bank can observe the previous winner's signal, i.e., the first-order statistics of rivals' signals $x^{(1)}$ from the first-period winning interest rate. Each bank updates their belief by this additional information. The posterior belief about the success probability of the potential borrower, $Prob(v = v_h | s_i, x^{(1)})$, is equal to:

$$v_1(s_i, x^{(1)}) = \frac{\mu(s_i) f(x^{(1)} | v_h) F(x^{(1)} | v_h)^{n-2}}{\mu(s_i) f(x^{(1)} | v_h) F(x^{(1)} | v_h)^{n-2} + (1 - \mu(s_i)) f(x^{(1)} | v_l) F(x^{(1)} | v_l)^{n-2}}.$$

We focus on the symmetric equilibrium with a bid function that is strictly decreasing in the private signal, s_i . The expected payoff for Bank i when it got a private signal s_i at date 0 and pretends to have gotten a signal y is equal to:

$$\pi_2(y; s_i, x^{(1)}) = E[(\min\{v, R_2(y; x^{(1)})\} - I) \mathbf{1}_{y > x^{(2)}} | x^{(1)}, s_i],$$

where $x^{(2)}$ is the second-highest rival's signal. Since the probability to win, given the first-order statistic among rival signals $x^{(1)}$, is

$$\frac{F^{n-2}(y|v)}{F^{n-2}(x^{(1)}|v)},$$

we can explicitly write down $\pi_2(y; s_i, x^{(1)})$ as follows:

$$(R_2(y; x^{(1)}) - I) \frac{F^{n-2}(y|v_h)}{F^{n-2}(x^{(1)}|v_h)} v_1(s_i, x^{(1)}) - \frac{F^{n-2}(y|v_l)}{F^{n-2}(x^{(1)}|v_l)} I (1 - v_1(s_i, x^{(1)})). \quad (5)$$

From the revelation principle, the next two conditions characterize the bid function of each bank $R_2(s_i; x^{(1)})$ in the symmetric equilibrium:

$$\left. \frac{\partial \pi_2(y; s_i, x^{(1)})}{\partial y} \right|_{y=s_i} = 0, \quad \forall s_i \in [\underline{s}, \bar{s}], \quad \forall i, \quad (6)$$

$$\pi_2(s_i; s_i, x^{(1)}) \geq 0, \quad \forall s_i \in [\underline{s}, \bar{s}], \quad \forall i, \quad (7)$$

Equation (6) is the incentive compatibility condition, which characterizes the equilibrium bidding in the direct mechanism. Equation (7) requires that a bank participating in the competition must expect to get a nonnegative return from bidding. If a bank draws a very low signal, it would like to quote a high interest rate to cover the high default cost due to the high default probability. However, the interest rate is, at most, v_h , the highest possible revenue from the project, because of the limited liability. Therefore, a bank refuses to lend if the signal is too low to cover the default cost. We denote the threshold of this screening in the second competition by s_{*2} . This threshold is defined implicitly by:

$$\pi_2(s_{*2}; s_{*2}, x^{(1)}) = 0, \quad (8)$$

$$R_2(s_{*2}, x^{(1)}) = v_h. \quad (9)$$

When a bank draws s_{*2} and wins in the competition, it is the only bidder. Therefore, it will quote $R_2(s_{*2}, x^{(1)}) = v_h$ in Eq. (9). These two conditions can be compiled into the following equation:

$$\frac{v_h - I}{I} = \frac{F^{n-2}(s_{*2}|v_l) f(x^{(1)}|v_l)}{F^{n-2}(s_{*2}|v_h) f(x^{(1)}|v_h)} \frac{1 - \mu(s_{*2})}{\mu(s_{*2})}. \quad (10)$$

It is possible to show that there exists a unique s_{*2} if the highest possible return from the project, v_h , is in a moderate range.

Lemma 1 (Existence of a unique s_{*2}). A unique $s_{*2} \in (\underline{s}, \bar{s})$ satisfying Eq. (10) exists if

$$\frac{1 - \gamma}{\gamma} \frac{f(\underline{s}|v_l)}{f(\underline{s}|v_h)} \frac{f(\bar{s}|v_l)}{f(\bar{s}|v_h)} > \frac{v_h - I}{I} > \frac{1 - \gamma}{\gamma} \frac{f(\bar{s}|v_l)}{f(\bar{s}|v_h)}. \quad (11)$$

Proof. See Appendix A.

We assume this condition holds in later analyses.

Since $\pi_2(s_i; s_i, x^{(1)})$ is strictly increasing in s_i under Eq. (6),⁵ conditions (8) and (9) are equivalent to Eq. (7). We can get a bid function for each bank by solving the differential equation (6) under the boundary condition (9).

3.1.2. No bank lent in the previous period

In this case, each bank knows that every rival's signal is less than the screening threshold in the first competition, s_{*1} . Therefore, each bank updates its belief about the subjective success

⁵ $\frac{d\pi_2(s_i; s_i, x^{(1)})}{ds_i} = \frac{d\pi_2(x; s_i, x^{(1)})}{dx} \Big|_{s_i=x} + \frac{d\pi_2(s_i; x, x^{(1)})}{dx} \Big|_{s_i=x} = \frac{d\pi_2(s_i; x, x^{(1)})}{dx} \Big|_{s_i=x}$ by the envelope theorem and Eq. (6). By assumption (1), $\frac{d\pi_2(s_i; x, x^{(1)})}{dx} \Big|_{s_i=x} > 0$.

probability of the firm to:

$$v_0(s_i, x^{(1)}) = \frac{\mu(s_i) F(s_{*1}|v_h)^{n-1}}{\mu(s_i) F(s_{*1}|v_h)^{n-1} + (1 - \mu(s_i)) F(s_{*1}|v_l)^{n-1}}.$$

The expected payoff for Bank i when it got private signal s_i and pretends to have gotten signal y is:

$$(R_2(y; s_{*1}) - I) \frac{F^{n-1}(y|v_h)}{F^{n-1}(s_{*1}|v_h)} v_0(s_i, s_{*1}) - I \frac{F^{n-1}(y|v_l)}{F^{n-1}(s_{*1}|v_l)} I (1 - v_0(s_i, s_{*1})). \quad (12)$$

From the same reasoning as in the previous section, the screening threshold is implicitly defined by:

$$\pi_2(s_{*2}; s_{*2}, s_{*1}) = 0, \quad (13)$$

$$R_2(s_{*2}, s_{*1}) = v_h. \quad (14)$$

These conditions can be compiled into the equation:

$$\frac{v_h - I}{I} = \frac{F^{n-1}(s_{*2}|v_l)}{F^{n-1}(s_{*2}|v_h)} \frac{1 - \mu(s_{*2})}{\mu(s_{*2})}. \quad (15)$$

This is exactly the same as Eq. (20) in Lemma 2 in the next section, which defines the threshold in the first competition s_{*1} . Therefore, nobody bids in the second competition if nobody lends in the first competition.

Proposition 1 (Equilibrium bid $R_2(s_i, x^{(1)})$). *If assumptions (1), (2), and (11) are satisfied and if there exists a symmetric equilibrium in the first competition, there exists a unique symmetric equilibrium in the second competition. In the equilibrium, each bank takes the following bidding strategy:*

- (1) *If a rival bank lent in the first competition at the interest rate of $R_1(x^{(1)})$, Bank i ($i = 1, \dots, n - 1$) bids:*

$$R_2(s_i, x^{(1)}) = v_h \frac{F^{n-2}(s_{*2}|v_h)}{F^{n-2}(s_i|v_h)} + \int_{s_{*2}}^{s_i} \left\{ K_2(t) \frac{F^{n-2}(t|v_h)}{F^{n-2}(s_i|v_h)} \right\} dt, \\ \text{where } K_2(t) = I(n - 2) \frac{f(t|v_h)}{F(t|v_h)} \left\{ 1 + \frac{1 - v_1(t, x^{(1)})}{v_1(t, x^{(1)})} \frac{f(t|v_l)}{f(t|v_h)} \frac{F^{n-2}(t|v_l)}{F^{n-2}(t|v_h)} \right\}, \quad (16)$$

which is strictly decreasing in s_i and $x^{(1)}$.

- (2) *If no bank lent in the first competition, no bank participates in the second competition.*

Proof. See Appendix A.

The more optimistic the winner's signal in the first competition is, the weaker the winner's curse for rival banks is. Consequently, the bid function decreases in the first winner's signal $x^{(1)}$. The information that nobody was willing to lend in the first competition works as a bad signal about the success probability of the borrower and makes every bank reluctant to lend in the second competition.

3.2. First competition

Given the symmetric equilibrium strategy in the second competition, each bank determines its bid by maximizing its expected payoff in the first competition conditional on the private signal s_i at date 0. The expected return when it got a private signal s_i and pretends to have gotten x is:

$$\pi_1(x; s_i) = E[(\min\{v, R_1(x)\} - I)\mathbf{1}_{x > x^{(1)}} | s_i] + E[(\min\{v, R_2(s_i, x^{(1)})\} - I)\mathbf{1}_{x^{(2)} < s_i, x < x^{(1)}} | s_i]. \quad (17)$$

The first term is the expected return from the first competition. The second term is the expected return in the second competition when it lost in the first period. The incentive compatibility condition is:

$$\left. \frac{\partial \pi_1(x, s_i)}{\partial x} \right|_{x=s_i} = 0, \quad \forall s_i \in [\underline{s}, \bar{s}] \text{ and } \forall i. \quad (18)$$

The individual rationality condition is:

$$\begin{aligned} & (\text{Expected return when Bank } i \text{ bids in the first period}) \\ & \geq (\text{Expected return when Bank } i \text{ does not bid in the first period}). \end{aligned} \quad (19)$$

The individual rationality condition determines the screening threshold in the first competition s_{*1} , which is derived in the next lemma.

Lemma 2 (Existence of a unique s_{*1}). *If assumption (11) in Lemma 1 is satisfied, there exists a unique equilibrium screening threshold $s_{*1} \in (\underline{s}, \bar{s})$ in the first competition. Bank i bids if and only if $s_i \geq s_{*1}$, where s_{*1} is implicitly defined by:*

$$\frac{v_h - I}{I} = \frac{F^{n-1}(s_{*2}|v_l)}{F^{n-1}(s_{*2}|v_h)} \frac{1 - \mu(s_{*1})}{\mu(s_{*1})}. \quad (20)$$

Proof. See Appendix A.

Bank i knows that it is the only bidder when it draws s_{*1} and wins. Therefore, it will quote the monopolistic interest rate, v_h . This defines the boundary condition:

$$R_1(s_{*1}) = v_h. \quad (21)$$

We can derive a bid function $R_1(s_i)$ by solving Eq. (18), which is a first-order linear differential equation with respect to R_1 , under the boundary condition (21).

Proposition 2 (Equilibrium bid $R_1(s_i)$). *If assumptions (1), (2), and (11) are satisfied, there exists a symmetric perfect Bayesian equilibrium in the first competition. At equilibrium, Bank i ($i = 1, \dots, n$) bids:*

$$\begin{aligned} R_1(s_i) &= v_h \frac{F^{n-1}(s_{*1}|v_h)}{F^{n-1}(s_i|v_h)} + \int_{s_{*1}}^{s_i} K_1(t) \frac{F^{n-1}(t|v_h)}{F^{n-1}(s_i|v_h)} dt, \\ \text{where } K_1(t) &= (n-1) \frac{f(t|v_h)}{F(t|v_h)} R_2(t, t). \end{aligned} \quad (22)$$

This is strictly decreasing in signal s_i .

Proof. See [Appendix A](#).

3.3. Comparison of bankers' willingness to lend in the two competitions

It is possible to show that banks are more willing to lend in the second competition than in the first competition once a rival lends in the first competition. The creditworthiness test of each bank is looser, and each interest rate bid is lower in the second competition than in the first competition. Banks that lost in the first competition are inspired by the information implied in the winning interest rate in the first competition and become more aggressive. In other words, they decrease the weight on their own private information s_i and increase the weight on the public information $R_1(x^{(1)})$ in their second-period decision making. This phenomenon is similar to the “information cascade” ([Bikhchandani et al., 1992](#)), but it is different in the sense that learning itself does not yield any economic inefficiency.⁶ The next two propositions summarize the results.

Proposition 3. *If a bank lends to a firm in the first competition, each bank bids more aggressively in the second competition than in the first competition, i.e., $R_1(s_i) > R_2(s_i, x^{(1)})$ in the symmetric perfect Bayesian equilibrium.*

Proof. The incentive compatibility condition (18) in the first period is:

$$\frac{\partial R_1(s_i)}{\partial s_i} = (n-1) \frac{f(s_i|v_h)}{F(s_i|v_h)} (R_2(s_i, s_i) - R_1(s_i)).$$

$R_2(s_i, s_i) < R_1(s_i)$ must hold since $\partial R_1(s_i)/\partial s_i$ is negative from the second statement in [Proposition 2](#).⁷ Furthermore, $R_2(s_i, x^{(1)})$ decreases in $x^{(1)}$ and $s_i < x^{(1)}$. Therefore, $R_1(s_i) > R_2(s_i, x^{(1)})$. $\square$

Proposition 4. *If at least one bank lends to a firm in the first competition, the creditworthiness test of each bank is looser in the second competition than in the first competition, i.e., $s_{*2} < s_{*1}$ in the symmetric Bayesian equilibrium.*

Proof. From [Eq. \(10\)](#) and [Eq. \(20\)](#), it is sufficient to show that

$$\frac{f(x^{(1)}|v_l)/F(s_i|v_l)}{f(x^{(1)}|v_h)/F(s_i|v_h)} < 1.$$

⁶ [Gale \(1996\)](#) and [Vives \(1996\)](#) identify the sufficient conditions for the existence of an inefficient information cascade, (i) the action space is smaller than the signal space, e.g., the action space is discrete, while the signal space is a continuum, (ii) belief is bounded, i.e., nobody knows that a certain event occurs for sure.

⁷ We assume that the discount factor is 1 in this proof. If we assume that the discount factor is $\delta < 1$, then the FOC in the proof is:

$$\frac{\partial R_1(s_i)}{\partial s_i} = (n-1) \frac{f(s|v_h)}{F(s_i|v_h)} (\delta R_2(s_i, s_i) - R_1(s_i)).$$

The above result still holds if δ is close to 1. In other words, this proposition holds if other investment opportunities are less profitable. A discount factor does not affect [Proposition 4](#).

Indeed, this holds since

$$\frac{f(x^{(1)}|v_l)/F(s_i|v_l)}{f(x^{(1)}|v_h)/F(s_i|v_h)} < \frac{f(s_i|v_l)/F(s_i|v_l)}{f(s_i|v_h)/F(s_i|v_h)} < 1. \quad (23)$$

The first inequality comes from assumption (1) and the observation that the participants in the second period have a more pessimistic signal than the winner in the first period, i.e., $s_i < x^{(1)}$. The second inequality comes from inequality (4). Therefore, the right-hand side in Eq. (10) is smaller than that of Eq. (20). Since the right-hand side of each equation decreases in s_i by the inequality (4) and assumption (1), s_{*1} must be greater than s_{*2} . $\square$

Once a rival bank lends to a firm, the winning bid or the equilibrium interest rate reveals the winner's private information that reflects the most optimistic opinion about the firm. The other banks update their belief about the firm by using this additional public information. Furthermore, the number of bidders decreases by the unit supply assumption that reflects the banker's incentive to limit their exposure to a single risk factor. The winner's curse for the losers is reduced by these two factors.

The winner's curse for Bank i can be restated as the probability that the firm is not successful, given that Bank i wins the lending competition. In the first competition, this probability is equal to:

$$\frac{(1 - \mu(s_i))F^{n-1}(s_i|v_l)}{\mu(s_i)F^{n-1}(s_i|v_h) + (1 - \mu(s_i))F^{n-1}(s_i|v_l)}. \quad (24)$$

In the second competition, when a rival bank won in the first competition, this probability is equal to:

$$\begin{aligned} & \frac{(1 - v_1(s_i, x^{(1)})) \frac{F^{n-2}(s_i|v_l)}{F^{n-2}(x^{(1)}|v_l)}}{v_1(s_i, x^{(1)}) \frac{F^{n-2}(s_i|v_h)}{F^{n-2}(x^{(1)}|v_h)} + (1 - v_1(s_i, x^{(1)})) \frac{F^{n-2}(s_i|v_l)}{F^{n-2}(x^{(1)}|v_l)}} \\ &= \frac{(1 - \mu(s_i))L(s_i, x^{(1)})F^{n-1}(s_i|v_l)}{\mu(s_i)F^{n-1}(s_i|v_h) + (1 - \mu(s_i))L(s_i, x^{(1)})F^{n-1}(s_i|v_l)}, \end{aligned} \quad (25)$$

where

$$L(s_i, x^{(1)}) = \frac{f(x^{(1)}|v_l)F(s_i|v_l)}{f(x^{(1)}|v_h)F(s_i|v_h)}.$$

The winner's curse is weaker in the second period than in the first period, since $L(s_i, x^{(1)})$ is smaller than 1 from inequality (23). This difference is due to the losers' learning from the former winning bid and the decrease in the number of bidders in the second competition.

3.4. Welfare analysis

Social welfare depends only on the strength of the creditworthiness test of each bank since a promised interest is merely a transfer within the economy. If the test is too strict, too many potentially profitable projects are passed up. If the test is too loose, too many potentially unsuccessful projects are carried out. At this point, the creditworthiness test in the first competition is excessively strict, since we assume that the competitive banker cannot participate in the next competition once it wins. Each bank sets the level of creditworthiness and the bid in a myopic

way without considering the effect of informational externality from its own bid on the beliefs of rival banks, which affects the outcome of the second competition.

The outcome of the second competition in the previous section is economically efficient under the public information given at date 2. Instead of looking at the maximization of welfare itself, we will focus on the minimization of the social cost. At date 2, the available information is the prior belief γ and the winner's private signal $x_{(1)}$. The expected social cost is:

$$(1 - v^p) \frac{1 - F^{n-1}(s_{*2}|v_l)}{F^{n-1}(x^{(1)}|v_l)} I + v^p \frac{F^{n-1}(s_{*2}|v_h)}{F^{n-1}(x^{(1)}|v_h)} (v_h - I), \quad (26)$$

where

$$v^p = \frac{\gamma f(x^{(1)}|v_h) F^{n-1}(x^{(1)}|v_h)}{\gamma f(x^{(1)}|v_h) F^{n-1}(x^{(1)}|v_h) + (1 - \gamma) f(x^{(1)}|v_l) F^{n-1}(x^{(1)}|v_l)}.$$

The first term is the cost when a bank lends to an unsuccessful project. The second term is the cost when every bank ignores a potentially successful project. The first-order condition for minimizing this social cost with respect to s_{*2} is:

$$-(1 - \mu(s_{*2}^o)) f(x^{(1)}|v_l) F^{n-2}(s_{*2}^o|v_l) I + \mu(s_{*2}^o) f(x^{(1)}|v_l) F^{n-2}(s_{*2}^o|v_h) (v_h - I) = 0, \quad (27)$$

where s_{*2}^o is the socially optimal level of the screening.⁸ This is exactly the same as Eq. (10), which defines the competitive screening threshold s_{*2} . Therefore, s_{*2}^o is equal to s_{*2} . In other words, the optimism resulting from learning does not cause any economic inefficiency by itself. The next proposition summarizes the result.

Proposition 5. *The outcome of the second competition is socially efficient under public information that is available at date 2.*

4. Comparative static—competitiveness and the impact of learning

A comparative static with respect to the number of banks n can show that increased competition yields a stronger winner's curse. This is verified by the fact that the subjective success probability of a borrower conditional on winning in each competition, (24) and (25), decreases in n . Consequently, the screening in each competition becomes stricter (higher s_{*1} and s_{*2}) as the credit market becomes more competitive. The effect of learning from the winning interest rate is partially swamped by this increased winner's curse, i.e., the difference in the strictness of the creditworthiness test in the two competitions decreases in n .

Proposition 6 (Effects on screening thresholds). *Screening thresholds s_{*1} and s_{*2} increase in the number of banks n . The difference $s_{*1} - s_{*2}$ decreases in n .*

Proof. See Appendix A.

This result means that severe competition makes each bank more prudent because of the stronger winner's curse and slows down the price-seeking process. However, the model suggests

⁸ The sufficient condition for the minimization is satisfied since the left-hand side of the first-order condition (27) is positive if $s_{*2}^o > s_{*2}$ and the left-hand side is negative if $s_{*2}^o < s_{*2}$.

that a harsher competition makes the economy more vulnerable to the downside risk, although it makes each individual bank more prudent.

Proposition 7 (*Effects on the aggregate risk*). *The welfare cost of lending to a potentially bad project in the second competition,*

$$(1 - v^p) \frac{1 - F^{n-1}(s_{*2}|v_l)}{F^{n-1}(x^{(1)}|v_l)} I,$$

increases in the number of banks n .

Proof. We assume that n is a real number. This assumption does not cause any additional problems since the function to be differentiated with respect to n turns out to be monotonic in n (Seade, 1980). By taking the derivative with respect to n and applying the envelope theorem to Eq. (26) with respect to s_{*2} , it is readily shown that the first term of Eq. (26) increases in n . $\square$

Increased competition makes the banking sector provide firms with more loanable funds and exposes it to a larger risk, although each individual bank becomes more cautious due to the increased winner's curse.

5. Some robustness checks

In the previous sections, we assume that

- (1) the return from a project is perfectly serially correlated,
- (2) the return for a firm is not correlated with that of other firms, and
- (3) each bank makes, at most, one loan.

In this section, we examine how the propositions in the previous sections are robust against the modifications in these assumptions.

5.1. Risk structure

Serial correlation In the previous sections, we assumed the perfect serial correlation of the returns from a project, i.e., a project is successful in both periods or unsuccessful in both periods. The assumption seems too strong, but it turns out that dropping the assumption merely changes the main results quantitatively, not qualitatively. To see this point, we assume that the project in the second period is successful and yields v_h with probability p if the project in the first period is successful and that it fails and yields nothing with probability p if the project in the first period is unsuccessful. We assume that p is greater than one half, i.e., the return from the project in each period is positively correlated. If p is equal to 1, the problem returns to the problem that we have already solved.

Under this new assumption, the expected return for a bank from the second competition when it got a private signal s_i at date 0 and pretends to have gotten a signal y is:

$$\begin{aligned} & \frac{F^{n-2}(y|v_h)}{F^{n-2}(x^{(1)}|v_h)} \{ (R_2(y; x^{(1)}) - I) v_1(s_i, x^{(1)}) p - I(1 - v_1(s_i, x^{(1)}))(1 - p) \} \\ & + \frac{F^{n-2}(y|v_l)}{F^{n-2}(x^{(1)}|v_l)} \{ (R_2(y; x^{(1)}) - I) v_1(s_i, x^{(1)})(1 - p) - I(1 - v_1(s_i, x^{(1)}))p \}. \end{aligned} \quad (28)$$

The equation to implicitly define the threshold s_{*2} is:

$$\frac{v_h - I}{I} = \frac{(1 - p)f(x^{(1)}|v_h)F^{n-2}(s_{*2}|v_h)\mu(s_{*2}) + pf(x^{(1)}|v_l)F^{n-2}(s_{*2}|v_l)(1 - \mu(s_{*2}))}{pf(x^{(1)}|v_h)F^{n-2}(s_{*2}|v_h)\mu(s_{*2}) + (1 - p)f(x^{(1)}|v_l)F^{n-2}(s_{*2}|v_l)(1 - \mu(s_{*2}))}. \quad (29)$$

If p is equal to 1, this equation is identical to Eq. (10). The right-hand side of this equation increases in p if $(v_h - I)/I \geq 1$ and decreases in p if $(v_h - I)/I < 1$. In any case, the lower p or the serial correlation of less than one does not affect Proposition 4, since Eq. (20), which defines the screening threshold in the first competition s_{*1} , does not change by the new assumption and its right-hand side is greater than the right-hand side of Eq. (29) as long as the serial correlation is positive. In the case in which $(v_h - I)/I < 1$, the conclusion of Proposition 4 becomes quantitatively weaker since the lower serial correlation decreases the impact of learning from the previous interest rate on the loan approval probability. We reached the same conclusion about the interest rate difference in Proposition 3 by a similar analysis.

Cross-sectional correlation So far, we have considered a case in which banks learn about the creditworthiness of a firm through the outcome of past lending competition. If banks learn about an industry factor or a macroeconomic factor through the outcome of a past lending competition to a firm in an economy or in an industry, the second statement in Proposition 1 that no bank participates in the second lending competition if no banks lent in the first competition needs to be modified. To demonstrate this point, let us consider the economy which consists of n banks and two firms whose returns depend only on the macroeconomic factor and are perfectly correlated. Even if Firm 1 does not obtain a loan in the first competition, it could obtain one in the second competition if Firm 2 obtained one in the first competition and at least one bank drew a private signal about Firm 1 between s_{*1} and s_{*2} since the belief updating regarding the macroeconomic factor affects the creditworthiness tests of both firms in the same way. Other propositions are obviously robust against the introduction of a cross-sectional correlation.

5.2. Multi-unit demand sequential auction

The assumption that each bank can make, at most, one loan is implausible in the context of small business financing, where the amounts of each loan are usually tiny compared to the size of each bank. If we allow each bank to lend in both periods, the winning bank needs to choose whether to undertake transaction banking or relationship banking after the first lending. Relationship banking is a banking mode in which banks collect proprietary information about their borrowers through sequential transactions (Boot, 2000; Boot and Thakor, 2000). A typical example is a small business loan, which entails ex post monitoring or consulting by a bank after lending. Transaction banking is a banking mode in which banks aim primarily at the returns from one-shot transactions. An example is a mortgage loan, which rarely entails ex post monitoring by a bank. We can show that the propositions in the previous sections are still true in almost all cases as long as banks undertake transaction banking, while the validity of them is vague if banks undertake relationship banking.

If banks learn primarily about a macroeconomic factor or an industry factor from the past interest rate, then the previous propositions are valid to the extent that they are valid under transaction banking since it is hardly believable that a bank obtains some proprietary information

about a macroeconomic factor or an industry factor through a certain customer relationship. However, if banks learn primarily about the creditworthiness of each individual firm from the outcome of the past lending competition, the existence of relationship banking matters significantly. In the latter case, the applicability of the propositions in the previous sections is limited to credit markets in which transaction banking is predominant, as we have noted in the introduction. Examples include a market in which mortgage loans account for a larger part of aggregate loans or a market in which many foreign banks enter tentatively to earn some short-term profits without establishing customer relationships.

Transaction banking If the winning bank undertakes transaction banking, it does not have any private information since it does not obtain any additional information that is not accessible to rival banks and its private signal s_i has been revealed to rivals through the winning interest rate in the symmetric equilibrium in the first competition. Therefore, the winning bank takes a mixed strategy in the equilibrium in the second competition (otherwise, an infinite loop of undercutting occurs). We denote the cumulative distribution function of the interest rate bid by the winning bank by $H(R_w)$. In addition, the winning bank evaluates the value of the second loan less than the rival banks because of the winner's curse and the effect of learning by losing banks from the first interest rate. Therefore, we can show that the expected return for the first winning bank from the second loan is zero from the result in the first-price common value auction under asymmetric information (Engelbrecht-Wiggans et al., 1983). Because of this zero-profit result, the screening threshold in the first competition s_{*1} , which is defined by Eq. (20), is not affected by this modification.

In the equilibrium in the second competition, where banks behave symmetrically except for the first winner, the expected return for a bank who lost in the first competition and bids $R_l(s_i)$ is:

$$\left((1 - H(R_l))q + 1 - q \right) \times \left(\frac{F^{n-2}(s_i|v_h)}{F^{n-2}(x^{(1)}|v_h)} (R_l - I) v_1(s_i, x^{(1)}) - \frac{F^{n-2}(s_i|v_l)}{F^{n-2}(x^{(1)}|v_l)} I (1 - v_1(s_i, x^{(1)})) \right), \quad (30)$$

where q is the first winner's participation probability. The inside of the first parenthesis is the probability to win against the first winner. The inside of the second parenthesis is the expected return from winning against other rivals. We can derive the same equation as Eq. (10), which defines the screening threshold in the second competition s_{*2} , by setting expression (30) as equal to zero at $R_l = v_h$.

In short, Proposition 4 and the comparative static results related to this proposition are valid for the banks who lost in the first competition as long as the first winner undertakes transaction banking. As for the interest rate bid difference in Proposition 3, we cannot get a clear conclusion since the first winner, who continues to be one of the rivals in the second competition, is less aggressive than in the first competition due to the winner's curse and this can affect the bids of other banks upward.

Relationship banking The first winning bank chooses relationship banking if it expects to earn a positive return from the informational advantage in the second competition exceeding the costs to collect additional information that is not accessible to rivals (Sharpe, 1990; Rajan, 1992). The opportunity costs of not bidding in the first competition include the cost of missing this earning opportunity in future (Petersen and Rajan, 1995). The previous results, regarding both the interest rate difference and the screening threshold, turn out to be ambiguous in this case.

For example, if the return from relationship banking in the second competition after winning in the first competition is expected to be π_r (> 0) at the beginning of the first competition, then the screening threshold in the first competition is:

$$\frac{v_h - I}{I} = \frac{F^{n-1}(s_{*1}|v_l)}{F^{n-1}(s_{*1}|v_h)} \frac{1 - \mu(s_{*1})}{\mu(s_{*1})} - \frac{\pi_r}{I}. \quad (31)$$

The second term in the right-hand side is negative. Therefore, the screening threshold in the first competition in the economy with relationship banking is lower than that in the economy without relationship banking defined by Eq. (20). In the second competition, π_r becomes zero since the project ends at the end of the second period, while the rival banks become more willing to lend by learning from the winning interest rate. The validity of Proposition 4 depends on which of these two factors dominates. This depends on the endogenously derived expected return from the customer relationship and the underlying assumption, including the functional form of F and other parameter values. It is an empirical task to examine whether the learning effect is strong enough to make Proposition 4 valid even under relationship banking.

6. Empirical implications

The empirical predictions from the present study are as follows:

- (1) Banks tend to loosen their creditworthiness tests during an economic boom (Proposition 3). This tendency is weaker when more banks are operating or relationship banking predominates in a credit market (Proposition 6 and the previous section about relationship banking).
- (2) When the number of competing banks is larger in a credit market, the creditworthiness test of each individual bank is stricter (Proposition 6) while the likelihood for a firm to get a loan from the credit market is greater (Proposition 7).

The first statement in the first prediction is observationally equivalent to the prediction from the institutional memory loss hypothesis (Berger and Udell, 2004). The institutional memory loss hypothesis attributes the tendency of banks to ease their credit standards during an economic boom to “a proportional increase in officers that never experienced a loan bust” and “the atrophying skills of experienced officers as time passes since their last problem-loans experience” (Berger and Udell, 2004, p. 459), while our model attributes the tendency to the learning from a past winner. The impact of the institutional memory loss depends on an institutional structure, such as information processing structure and a human resource management system in each bank. In contrast, the effect of the learning from a winner depends on market structure, such as the number of rival banks and the prevalence of relationship banking. At this point, the second statement in the first prediction is useful to empirically tell apart these competing hypotheses. If the institutional memory loss is the dominant force to ease credit standards, a remedy must be prescribed to avoid the economic inefficiency due to the memory loss. If the learning from a winner is dominant, a remedy is not required from the welfare economic point of view since the easing of credit standards is interpreted as the result of an efficient price-seeking process (see Proposition 5).

An empirical examination of the second prediction has another important policy implication. It suggests that the government in an economy where financial liberalization is expected need to take a stricter prudential policy after liberalization than before if the government and the people in the economy are risk averse and prefer to limit a downside risk.

7. Conclusion

In this paper, we illustrate the process in which learning from the rival winner makes loan terms more favorable for a borrower at later stages in a credit market in which transaction banking is predominant. This process is a rational learning process among banks seeking a proper loan price and is not necessarily economically inefficient by itself. Increased competition attenuates the learning impact for each individual bank or slows down the loan price-seeking process, while it increases the aggregate risk taken by the entire banking sector unless the banking sector becomes fully informed about the borrowers. This result implies that the bank supervision needs to be particularly conservative after financial liberalization if a government prefers to maintain the aggregate risk at the level before liberalization.

Acknowledgments

I am grateful to Lee Branstetter, Eslyn Jean-Baptiste, Michael Riordan at Columbia University, Anjan Thakor at Washington University, Kozo Kunimune at the Institute of Developing Economies, Kazunari Ohashi at the Bank of Japan, and the anonymous referee for insightful and valuable comments. All remaining errors are mine.

Appendix A

Proof of Lemma 1. The right-hand side of Eq. (10) is monotone decreasing in s_{*2} from assumption (1). Therefore, there exists a unique s_{*2} if and only if the right-hand side is greater than the left-hand side at $\underline{s}$ and the right-hand side is smaller than the left-hand side at $\bar{s}$,

$$\begin{aligned} & \frac{1-\gamma}{\gamma} \frac{f(\underline{s}|v_l)}{f(\underline{s}|v_h)} \frac{f(x^{(1)}|v_l)}{f(x^{(1)}|v_h)} \left(\frac{F(\underline{s}|v_l)}{F(\underline{s}|v_h)} \right)^{n-1} \\ & > \frac{v_h - I}{I} > \frac{1-\gamma}{\gamma} \frac{f(\bar{s}|v_l)}{f(\bar{s}|v_h)} \frac{f(x^{(1)}|v_l)}{f(x^{(1)}|v_h)} \left(\frac{F(\bar{s}|v_l)}{F(\bar{s}|v_h)} \right)^{n-1}. \end{aligned}$$

Since $F(\underline{s}|v_h) = F(\underline{s}|v_l) = 0$ and $F(\bar{s}|v_h) = F(\bar{s}|v_l) = 1$,

$$\frac{1-\gamma}{\gamma} \left(\frac{f(\underline{s}|v_l)}{f(\underline{s}|v_h)} \right)^n \frac{f(x^{(1)}|v_l)}{f(x^{(1)}|v_h)} > \frac{v_h - I}{I} > \frac{1-\gamma}{\gamma} \frac{f(\bar{s}|v_l)}{f(\bar{s}|v_h)} \frac{f(x^{(1)}|v_l)}{f(x^{(1)}|v_h)}.$$

The first term comes from the L'Hopital's rule. If $s_i \rightarrow \bar{s}$, then $x^{(1)} \rightarrow \bar{s}$. Furthermore, $\frac{f(x^{(1)}|v_l)}{f(x^{(1)}|v_h)}$ is decreasing in $x^{(1)}$ and assumption (1) implies $\frac{f(\bar{s}|v_l)}{f(\bar{s}|v_h)} < 1$ and $\frac{f(\underline{s}|v_l)}{f(\underline{s}|v_h)} > 1$. Therefore, the next inequality implies the above inequality:

$$\frac{1-\gamma}{\gamma} \frac{f(\underline{s}|v_l)}{f(\underline{s}|v_h)} \frac{f(\bar{s}|v_l)}{f(\bar{s}|v_h)} > \frac{v_h - I}{I} > \frac{1-\gamma}{\gamma} \frac{f(\bar{s}|v_l)}{f(\bar{s}|v_h)}. \quad \square$$

Proof of Proposition 1. (1) *Equilibrium bid when at least one bank lent in the first competition:* The first-order condition (6) in this case is:

$$\begin{aligned} \frac{\partial R_2}{\partial y} + (R_2(y; x^{(1)}) - I)(n-2) \frac{f(y|v_h)}{F(y|v_h)} \\ - I(n-2) \frac{f(y|v_l)}{F(y|v_l)} \frac{F^{n-2}(y|v_l)}{F^{n-2}(y|v_h)} \frac{1 - \mu(s_i)}{\mu(s_i)} \frac{f(x^{(1)}|v_l)}{f(x^{(1)}|v_h)} = 0, \quad \text{at } y = s_i. \end{aligned} \quad (32)$$

Solving this first-order linear differential equation with the boundary condition (9) gives the bid function in the proposition. $\square$

(2) *Sufficient condition*: It is sufficient to show that the payoff function $\pi_2(y; s_i, x^{(1)})$ satisfies the single crossing condition under the first-order condition (6), i.e.,

$$\frac{\partial^2 \pi_2(y; s_i, x^{(1)})}{\partial y \partial s_i} \geq 0, \quad \forall s_i, \forall y.$$

Under the first-order condition (6),

$$\frac{\partial^2 \pi_2(y; s_i, x^{(1)})}{\partial y \partial s_i} = I(n-2) f(y|v_l) f(x^{(1)}|v_l) F^{n-3}(y|v_l) \frac{\mu'(s_i)}{\mu(y)}.$$

From assumption (1), this is positive. $\square$

(3) *Strict decreasing of $R_2(s_i; x^{(1)})$ in s_i* . $\pi_2(s_i; s_i, x^{(1)})$ is strictly increasing in s_i under the first-order condition (6) by the envelope theorem and is monotone increasing in R_2 . Therefore, by the implicit function theorem:

$$\frac{dR_2}{ds_i} = \frac{\partial \pi_2 / \partial s_i}{\partial \pi_2 / \partial R_2} < 0. \quad \square$$

(4) *Monotone decreasing of $R_2(s_i; x^{(1)})$ in $x^{(1)}$* . Differentiating the equilibrium bid (16) with respect to $x^{(1)}$ gives:

$$\begin{aligned} \frac{\partial R_2}{\partial x^{(1)}} &= \frac{\partial R_2}{\partial s_{*2}} \frac{\partial s_{*2}}{\partial x^{(1)}} + I \frac{\partial}{\partial x^{(1)}} \\ &\times \left(\frac{f(x^{(1)}|v_l)}{f(x^{(1)}|v_h)} \right) \int_{s_{*2}}^{s_i} \frac{1 - \mu(t)}{\mu(t)} \frac{f(u|v_l)}{F(u|v_l)} \frac{F^{n-2}(u|v_l)}{F^{n-2}(u|v_h)} \frac{F^{n-2}(u|v_h)}{F^{n-2}(s_i|v_h)} du. \end{aligned}$$

The first term is equal to 0 from the definition of s_{*2} , Eq. (10). The second term is negative from assumption (1). $\square$

Proof of Lemma 2. (1) *Derivation of the threshold s_{*1}* . The expected return for Bank i when it bids given that the rival banks participate in the bidding when their signal is greater than or equal to s_{*1} is:

$$\begin{aligned} F^{n-1}(s_i|v_h) (R_1(s_i) - I) \mu(s_i) - I F^{n-1}(s_i|v_l) (1 - \mu(s_i)) \\ + (n-1) (1 - F(s_i|v_h)) F^{n-2}(s_i|v_h) \left(\int_{s_i}^{\bar{s}} R_2(s_i; w) \frac{f(w|v_h)}{1 - F(s_i|v_h)} dw - I \right) \mu(s_i) \\ - (n-1) (1 - F(s_i|v_l)) F^{n-2}(s_i|v_l) I (1 - \mu(s_i)). \end{aligned}$$

The first term is the expected return from the first bid. The second and third term is the expected return from the second bid. If it does not bid in the first competition, the expected return is:

$$\begin{aligned}
 & (F^{n-1}(s_i|v_h) - F^{n-1}(s_{*1}|v_h)) \\
 & \times \left(\int_{s_{*1}}^{s_i} R_2(s_i, w) \frac{(n-1)F^{n-2}(w|v_h)(1-F(w|v_h))f(w|v_h)}{F^{n-1}(s_i|v_h) - F^{n-1}(s_{*1}|v_h)} dw - I \right) \mu(s_i) \\
 & - I(F^{n-1}(s_i|v_l) - F^{n-1}(s_{*1}|v_l))(1 - \mu(s_i)) \\
 & + (n-1)(1 - F(s_i|v_h))F^{n-2}(s_i|v_h) \left(\int_{s_i}^{\bar{s}} R_2(s_i; w) \frac{f(w|v_h)}{1 - F(s_i|v_h)} dw - I \right) \mu(s_i) \\
 & - (n-1)^2(1 - F(s_i|v_l))^2 F^{n-2}(s_i|v_l) I(1 - \mu(s_i))^2 \\
 & + \max[0, F^{n-1}(s_{*1}|v_h)(v_h - I)\mu(s_i) - F^{n-1}(s_{*1}|v_l)I(1 - \mu(s_i))].
 \end{aligned}$$

The first, second, and third lines are the expected return for Bank i from the second competition when it does not bid although it has the highest private signal among its rivals. In this case, the rival with the second highest signal wins in the first competition. Therefore, the expected value must be calculated by using the probability distribution function of the second order statistic. The fourth and fifth lines are the expected return when it has the second highest signal. The last line is the expected return when no rivals bid in the first competition. In this case, Bank i bids the monopolistic rate v_h as long as it yields a positive return since the rivals do not participate in the second competition from the analysis in Section 3.1.2.

The difference of these two expected returns after substituting Eq. (22) is:

$$\mu(s_i) \int_{s_{*1}}^{s_i} K_1(t) \frac{F^{n-1}(t|v_h)}{F^{n-1}(s_i|v_h)} dt,$$

if $F^{n-1}(s_{*1}|v_h)(v_h - I)\mu(s_i) - F^{n-1}(s_{*1}|v_h)I(1 - \mu(s_i))$ is positive. Otherwise, it is:

$$\begin{aligned}
 & \mu(s_i) \int_{s_{*1}}^{s_i} K_1(t) \frac{F^{n-1}(t|v_h)}{F^{n-1}(s_i|v_h)} dt \\
 & - (F^{n-1}(s_{*1}|v_h)(v_h - I)\mu(s_i) - F^{n-1}(s_{*1}|v_l)I(1 - \mu(s_i))).
 \end{aligned}$$

In order that this expression is greater than zero if and only if $s_i \geq s_{*1}$, it must be true that:

$$F^{n-1}(s_{*1}|v_h)(v_h - I)\mu(s_i) - F^{n-1}(s_{*1}|v_l)I(1 - \mu(s_i)) = 0 \quad \text{if } s_i = s_{*1}.$$

This equation gives Eq. (20), which defines s_{*1} . $\square$

(2) *Existence.* s_{*1} , which is defined by Eq. (20), exists if

$$\frac{1 - \gamma}{\gamma} \frac{f(\underline{s}|v_l)}{f(\underline{s}|v_h)} > \frac{v_h - I}{I} > \frac{1 - \gamma}{\gamma} \frac{f(\bar{s}|v_l)}{f(\bar{s}|v_h)}.$$

This condition is derived in the same logic as in the proof of Lemma 1. This inequality is implied by inequality (11). Therefore, assumption (11) is sufficient for the existence of s_{*1} . $\square$

Proof of Proposition 2. (1) *Bid function.* The expected return $\pi_1(x; s_i)$ for Bank i when it gets a private signal s_i and pretends to have got a signal x is:

$$\begin{aligned} \pi_1(x; s_i) &= F^{n-1}(x|v_h)(R_1(x) - I)\mu(s_i) - IF^{n-1}(x|v_l)(1 - \mu(s_i)) \\ &\quad + (n-1)(1 - F(x|v_h))F^{n-2}(s_i|v_h)\left(\int_x^{\bar{s}} R_2(s_i, w)\frac{f(w|v_h)}{1 - F(x|v_h)}dw - I\right)\mu(s_i) \\ &\quad - (n-1)(1 - F(x|v_l))F^{n-2}(s_i|v_l)I(1 - \mu(s_i)) \quad \text{if } x \geq s_i, \end{aligned} \quad (33)$$

$$\begin{aligned} \pi_1(x; s_i) &= F^{n-1}(x|v_h)(R_1(x) - I)\mu(s_i) - IF^{n-1}(x|v_l)(1 - \mu(s_i)) \\ &\quad + (n-1)(1 - F(s_i|v_h))F^{n-2}(s_i|v_h)\left(\int_{s_i}^{\bar{s}} R_2(s_i, w)\frac{f(w|v_h)}{1 - F(s_i|v_h)}dw - I\right)\mu(s_i) \\ &\quad - I(n-1)(1 - F(s_i|v_l))F^{n-2}(s_i|v_l)(1 - \mu(s_i)) \\ &\quad + \left(\int_x^{s_i} R_2(s_i, w)(n-1)f(w|v_h)F^{n-2}(w|v_h)dw\right)\mu(s_i) \\ &\quad - (F^{n-1}(s_i|v_h) - F^{n-1}(x|v_h))I\mu(s_i) \\ &\quad - (F^{n-1}(s_i|v_l) - F^{n-1}(x|v_l))I(1 - \mu(s_i)) \quad \text{if } x < s_i. \end{aligned} \quad (34)$$

The first two terms are the expected payoff in the first competition. The next two terms are the expected payoff when the bank loses in the first stage and wins in the second stage. We need to calculate the expected value with respect to the winner's signal in the first competition. The last two terms in Eq. (34) are the expected payoff when $x^{(1)} \in [x, s_i]$. In both cases, the first-order condition (18) is:

$$\frac{\partial R_1(s_i)}{\partial s_i} = (n-1)\frac{f(s_i|v_h)}{F(s_i|v_h)}(R_2(s_i, s_i) - R_1(s_i)). \quad (35)$$

By solving this linear differential equation with the boundary condition (21), we can get the equilibrium bid function (22). $\square$

(2) *Monotone decreasing in s_i .* By the implicit function theorem,

$$\frac{dR_1}{ds_i} = -\frac{\partial \pi_1 / \partial s_i}{\partial \pi_1 / \partial R_1}.$$

Obviously $\partial \pi_1 / \partial R_1 > 0$. Therefore, it is sufficient to show that $\partial \pi_1 / \partial s_i > 0$ in order to show $dR_1/ds_i < 0$. $\partial \pi_1(s_i; s_i)/\partial s_i$ is equal to:

$$\frac{\partial \pi_1(x; y, s_i)}{\partial x} \Big|_{x=y=s_i} + \frac{\partial \pi_1(x; y, s_i)}{\partial y} \Big|_{x=y=s_i} + \frac{\partial \pi_1(x; y, s_i)}{\partial s_i} \Big|_{x=y=s_i}. \quad (36)$$

By the envelop theorem, the first two terms are 0 under the two first-order conditions in the first and second competitions. The last term is positive. Therefore, $\partial \pi_1 / \partial s_i > 0$. $\square$

(3) *Sufficient condition.* If $x > s_i$,

$$\frac{\partial \pi_1}{\partial x} = (n-1)f(x|v_h)F^{n-2}(x|v_h)\mu(s_i)(R_1(x) - R_2(s_i, x))F^{n-1}(x|v_h)\mu(s_i)\frac{\partial R_1(x)}{\partial x}.$$

Substituting the first-order condition (35) at $s_i = x$ gives:

$$\frac{\partial \pi_1}{\partial x} = (n-1)(\pi_2(x; s_i, x) - \pi_2(s_i; s_i, x)).$$

This is negative since $\pi_2(y; s_i, x)$ is the maximum at $y = s_i$ as we have seen in the proof of Proposition 1. Therefore, Bank i does not have any incentive to disguise upward.

If $x \leq s_i$,

$$\begin{aligned} \frac{\partial^2 \pi_1(x; s_i)}{\partial x \partial s_i} &= (R_2(x, x) - R_2(s_i, x))(n-1)f(x|v_h)F^{n-2}(x|v_h)\mu'(s_i) \\ &\quad - \frac{\partial^2 R_2(s_i, s_i)}{\partial s_i} (n-1)f(x|v_h)F^{n-2}(x|v_h)\mu(s_i), \end{aligned}$$

after substituting the two first-order conditions in the first and second competitions. This is non-negative since $R_2(s_i, x)$ is decreasing in s_i from Proposition 1 and $x \leq s_i$. The single crossing property holds for all s_i and all x . $\square$

Proof of Proposition 6 (First statement). The right-hand sides of Eqs. (10) and (20) are increasing in n and decreasing in s_{*1} and s_{*2} , respectively.

(Second statement) In this proof, we assume that n is a real number. This assumption does not cause any problem since the function in the problem turned out to be a monotone decreasing in n (Seade, 1980). By applying the implicit function theorem to Eqs. (10) and (20), we get:

$$\begin{aligned} \frac{\partial s_{*1}}{\partial n} &= - \frac{\log F(s_{*1}|v_l) - \log F(s_{*1}|v_h)}{\frac{\partial}{\partial s_{*1}} \log \frac{F^{n-1}(s_{*1}|v_l)}{F^{n-1}(s_{*1}|v_h)} + \frac{\partial}{\partial s_{*1}} \log \frac{1-\mu(s_{*1})}{\mu(s_{*1})}}, \\ \frac{\partial s_{*2}}{\partial n} &= - \frac{\log F(s_{*2}|v_l) - \log F(s_{*2}|v_h)}{\frac{\partial}{\partial s_{*2}} \log \frac{F^{n-2}(s_{*2}|v_l)}{F^{n-2}(s_{*1}|v_h)} + \frac{\partial}{\partial s_{*2}} \log \frac{1-\mu(s_{*2})}{\mu(s_{*2})}}. \end{aligned}$$

At any point of $s_{*1} = s_{*2} = s$, the first derivative is smaller than the latter by inequality (4). This means $s_{*1} - s_{*2}$ is decreasing in n . $\square$

References

- Berger, A., Udell, G., 2004. The institutional memory hypothesis and the procyclicality of bank lending behavior. *J. Finan. Intermediation* 13, 458–495.
- Bikhchandani, S., Hirshleifer, D., Welch, I., 1992. A theory of fads, fashion, custom, and cultural change as informational cascades. *J. Polit. Economy* 100, 992–1026.
- Boot, A., 2000. Relationship banking: What do we know? *J. Finan. Intermediation* 9, 7–25.
- Boot, A., Thakor, A., 2000. Can relationship banking survive competition? *J. Finance* 55, 679–713.
- Broecker, T., 1990. Credit-worthiness tests and interbank competition. *Econometrica* 58, 429–452.
- Caprio, G., Klingebiel, D., 2000. Bank insolvency: Bad luck, bad policy, or bad banking? In: Papadimitriou, D. (Ed.), *Modernizing Financial Systems*. St. Martin's, pp. 267–301.
- Coval, J., Thakor, A., 2005. Financial intermediation as a beliefs-bridge between optimists and pessimists. *J. Finan. Econ.* 75, 535–569.
- Engelbrecht-Wiggans, R., Milgrom, P., Weber, R., 1983. Competitive bidding and proprietary information. *J. Math. Econ.* 11, 161–169.
- Gale, D., 1996. What have we learned from social learning? *Europ. Econ. Rev.* 40, 617–628.
- Hoshi, T., Kashyap, A., 1999. The Japanese banking crisis: Where did it come from and how will it end? In: Bernanke, B., Rotemberg, J. (Eds.), *NBER Macroeconomics Annual*. MIT Press, pp. 129–201.

- Kaminsky, G., Reinhart, C., 1999. The twin crises: The causes and banking and balance-of-payments problems. *Amer. Econ. Rev.* 89, 473–500.
- Lauridsen, L., 1998. The financial crisis in Thailand: Causes, conduct and consequences? *World Devel.* 26, 1575–1591.
- Manove, M., Padilla, J., 1999. Banking (conservatively) with optimists. *RAND J. Econ.* 30, 324–350.
- Milgrom, P., Weber, R., 1982. A theory of auctions and competitive bidding, II. Chapter 8 in: Klemperer, P. (Ed.), *The Economic Theory of Auctions*. Edward Elgar.
- Ongena, S., Smith, D., Michalsen, D., 2003. Firms and their distressed banks: Lessons from the Norwegian banking crisis. *J. Finan. Econ.* 67, 81–112.
- Ortega-Reichert, A., 1968. A sequential game with information flow. Chapter 6 in: Klemperer, P. (Ed.), *The Economic Theory of Auctions*. Edward Elgar.
- Petersen, M., Rajan, R., 1995. The effect of credit market competition on lending relationships. *Quart. J. Econ.* 110, 407–443.
- Rajan, R., 1992. Insiders and outsiders: The choice between informed and arm's-length debt. *J. Finance* 47, 191–212.
- Riordan, M., 1993. Competition and bank performance: A theoretical perspective. In: Mayer, C., Vives, X. (Eds.), *Capital Markets and Financial Intermediation*. Cambridge Univ. Press, pp. 328–343.
- Seade, J., 1980. On the effects of entry. *Econometrica* 48, 479–490.
- Shaffer, S., 1998. The winner's curse in banking. *J. Finan. Intermediation* 7, 359–392.
- Sharpe, S., 1990. Asymmetric information, bank lending and implicit contracts: A stylized model of customer relationships. *J. Finance* 45, 1069–1087.
- Thakor, A., 1996. Capital requirements, monetary policy, and aggregate bank lending: Theory and empirical evidence. *J. Finance* 51, 279–324.
- Thakor, A., 2005. Do loan commitments cause overlending? *J. Money, Credit, Banking* 37, 1067–1099.
- Vives, X., 1996. Social learning and rational expectations. *Europ. Econ. Rev.* 40, 589–601.
- Williams, J., 1998. Resist market pressure to make 'ugly loans' (excerpts from a September 18 speech on credit risk by acting Comptroller of the Currency Julie L. Williams to members of the Bankers Roundtable). *The Amer. Banker*, October 3.